NAG Toolbox for MATLAB

f07hn

1 Purpose

f07hn computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive-definite band matrix of bandwidth $(2k_d + 1)$ and X and B are n by r matrices.

2 Syntax

```
[ab, b, info] = f07hn(uplo, kd, ab, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07hn uses the Cholesky decomposition to factor A as $A = U^{\mathrm{H}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{H}}$ if **uplo** = 'L', where U is an upper triangular band matrix, and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

If **uplo** = 'U', the upper triangle of A is stored.

If **uplo** = 'L', the lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **kd – int32 scalar**

k_d, the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0.

3: **ab**(**ldab**,$*$) **– complex array**

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the Hermitian band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

4: **b(ldb,∗) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **ab(ldab,∗) – complex array**

The first dimension of the array **ab** must be at least $\mathbf{kd} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **info** = 0, the triangular factor U or L from the Cholesky factorization $A = U^{\mathrm{H}}U$ or $A = LL^{\mathrm{H}}$ of the band matrix A, in the same storage format as A.

2: **b(ldb,∗) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If **info** = 0, the n by r solution matrix X.

3: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **nrhs_p**, 5: **ab**, 6: **ldab**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the leading minor of order i of A is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07hp is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04cf solves $Ax = b$ and returns a forward error bound and condition estimate. f04cf calls f07hn to solve the equations.

8 Further Comments

When $n \gg k$, the total number of floating-point operations is approximately $4n(k+1)^2 + 16nkr$, where k is the number of superdiagonals and r is the number of right-hand sides.

The real analogue of this function is f07ha.

9 Example

```
uplo = 'U';
kd = int32(1);
ab  =  [complex(0,  0),  complex(1.08,  -1.73),  complex(-0.04,  +0.29),
complex(-0.33, +2.24);
           complex(9.39,  +0),  complex(1.69,  +0),  complex(2.65,  +0),
complex(2.17, +0)];
b = [complex(-12.42, +68.42);
     complex(-9.93, +0.88);
     complex(-27.3, -0.01);
     complex(5.31, +23.63)];
[abOut, bOut, info] = f07hn(uplo, kd, ab, b)
```

```
abOut =
         0               0.3524 - 0.5646i  -0.0358 + 0.2597i  -0.2054 +
1.3942i
   3.0643            1.1167            1.6066            0.4289
bOut =
  -1.0000 + 8.0000i
   2.0000 - 3.0000i
  -4.0000 - 5.0000i
   7.0000 + 6.0000i
info =
```

0